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Numerical Analysis (10th Edition)

Chapter 9.3, Problem 3E Bookmark Show all steps: ON

Problem

Repeat Exercise 1 using the Inverse Power method.

Reference: Exercise 1

Find the first three iterations obtained by the Power method applied to the following matrices.

a. $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$; Use $\mathbf{x}^{(0)} = (1,-1,2)^t$.

b. $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$; Use $\mathbf{x}^{(0)} = (-1,0,1)^t$.

c. $\begin{bmatrix} 1 & -1 & 0 \\ -2 & 4 & -2 \\ 0 & -1 & 2 \end{bmatrix}$; Use $\mathbf{x}^{(0)} = (-1,2,1)^t$.

d. $\begin{bmatrix} 4 & 1 & 1 & 1 \\ 1 & 3 & -1 & 1 \\ 1 & -1 & 2 & 0 \\ 1 & 1 & 0 & 2 \end{bmatrix}$; Use $\mathbf{x}^{(0)} = (1,-2,0,3)^t$.

Step-by-step solution

Step 1 of 32

Objective is to determine the first three iterations obtained by the Inverse Power method applied to the given matrices with initial vector.

The following theory is required to understand the inverse power method.

Inverse Power Method:

It is the modification of the Power Method and yields the dominant eigenvalue and the associated eigenvector at a faster convergence.

This method is applied to determine the eigenvalue of a given matrix A that is closest to a specified number q.

Comment

Step 2 of 32

If $\lambda_1, \lambda_2, \ldots, \lambda_n$ are the eigenvalues with associated linearly independent eigenvectors $\{v^{(1)}, v^{(2)}, \ldots v^{(n)}\}$ of the matrix A then the eigenvalues of the matrix $(A-qI)^{-1}$ where, $q \neq \lambda_i$ for $i = 1,2,\ldots,n$ are given by $\frac{1}{\lambda_1 - q}, \frac{1}{\lambda_2 - q}, \ldots, \frac{1}{\lambda_n - q}$ with the same eigenvectors.

Apply the Power Method to the matrix $(A-qI)^{-1}$ as shown below.

$$y^{(m)} = (A-qI)^{-1} x^{(m-1)}, \quad \mu^{(m)} = y^{(m)}_{p_{m-1}} \text{ and } x^{(m)} = \left(\frac{y^{(m)}}{y^{(m)}_{p_m}}\right)$$

Here the following situation arises.

$$\begin{aligned} \mu^{(m)} &= y^{(m)}_{p_{m-1}} \\ &= \frac{y^{(m)}_{p_{m-1}}}{x^{(m)}_{p_{m-1}}} \\ &= \left(\frac{\sum_{j=1}^{n} \beta_j \left(\frac{1}{\lambda_j - q}\right)^m v^{(j)}_{p_{m-1}}}{\sum_{j=1}^{n} \beta_j \left(\frac{1}{\lambda_j - q}\right)^{m-1} v^{(j)}_{p_{m-1}}} \right) \end{aligned}$$

Comment

Step 3 of 32

Continuation of the above

$$\mu^{(m)} = \left(\frac{1}{\lambda_k - q}\right) \left(\frac{\beta_k v^{(j)}_{p_{m-1}} + \sum_{j=1}^{n} \beta_j \left(\frac{\lambda_k - q}{\lambda_j - q}\right)^m v^{(j)}_{p_{m-1}}}{\beta_k v^{(j)}_{p_{m-1}} + \sum_{j=1}^{n} \beta_j \left(\frac{\lambda_k - q}{\lambda_j - q}\right)^{m-1} v^{(j)}_{p_{m-1}}} \right), \text{ where } q = \left\{ \frac{\left(x^{(0)}\right)^t A x^{(0)}}{\left(x^{(0)}\right)^t x^{(0)}} \right\}$$

Where at each step, p_m is used to represent the smallest integer for which $\left|y^{(m)}_{p_m}\right| = \left\|y^{(m)}\right\|_\infty$

Finally the Inverse Power method concludes as shown below

$$\mu^{(m)} = \left(\frac{1}{\lambda_k - q}\right) \left(\frac{\beta_k v^{(j)}_{p_{m-1}} + \sum_{j=1}^{n} \beta_j \left(\frac{\lambda_k - q}{\lambda_j - q}\right)^m v^{(j)}_{p_{m-1}}}{\beta_k v^{(j)}_{p_{m-1}} + \sum_{j=1}^{n} \beta_j \left(\frac{\lambda_k - q}{\lambda_j - q}\right)^{m-1} v^{(j)}_{p_{m-1}}} \right)$$

$$\lim_{m\to\infty} \mu^{(m)} = \lim_{m\to\infty} \left\{ \left(\frac{1}{\lambda_k - q}\right) \left(\frac{\beta_k v^{(j)}_{p_{m-1}} + \sum_{j=1}^{n} \beta_j \left(\frac{\lambda_k - q}{\lambda_j - q}\right)^m v^{(j)}_{p_{m-1}}}{\beta_k v^{(j)}_{p_{m-1}} + \sum_{j=1}^{n} \beta_j \left(\frac{\lambda_k - q}{\lambda_j - q}\right)^{m-1} v^{(j)}_{p_{m-1}}} \right) \right\}$$

$$\mu^{(m)} = \left(\frac{1}{\lambda_k - q}\right) \quad \left\{ \text{since, } \lim_{m\to\infty} \left(\frac{\lambda_k - q}{\lambda_j - q}\right)^m = 0 \text{ and } \beta_k \neq 0 \right\}$$

$$\lambda_k \approx q + \frac{1}{\mu^{(m)}}$$

Thus, the choice of q determines the convergence, provided that $\frac{1}{\lambda_k - q}$ is a unique dominant eigenvalue of $(A-qI)^{-1}$

Comment

Step 4 of 32

(a)

Consider the matrix and initial vector,

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \text{ and } \mathbf{x}^{(0)} = (1,-1,2)^t$$

Determine the first three iterations obtained by the Inverse Power method applied to the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$ with $x^{(0)} = (1,-1,2)^t$ by using Maple technology as shown below.

Activate linear algebra package in maple by using with(LinearAlgebra) package.

Comment

Step 5 of 32

Input the given matrix A and the initial vector $x^{(0)}$

```
> with(LinearAlgebra) :
> A := Matrix([[2, 1, 1], [1, 2, 1], [1, 1, 2]])
```

$$A := \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

```
> x0 := ⟨1,-1, 2⟩
```

$$x0 := \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

Comment

Step 6 of 32

Find $q = \left\{ \frac{\left(x^{(0)}\right)^t A x^{(0)}}{\left(x^{(0)}\right)^t x^{(0)}} \right\}$ in continuation of the above maple commands.

```
> w := Transpose(x0)
```

$w := [\,1 \;\; -1 \;\; 2\,]$

```
> q := w.A.x0 / w.x0
```

$q := \frac{5}{3}$

```
> evalf(q)
```

1.666666667

Comment

Step 7 of 32

Find $y^{(1)} = (A-qI)^{-1} x^{(0)}$ and $\left\|y^{(1)}\right\|_\infty = y^{(1)}_1$ in continuation of the above maple commands.

```
> i := IdentityMatrix(3)
```

$$i := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

```
> y1 := (A - q·i)^(-1).x0
```

$$y1 := \begin{bmatrix} -\frac{3}{14} \\ \frac{39}{14} \\ -\frac{12}{7} \end{bmatrix}$$

```
> Norm(y1, infinity)
```

$\frac{39}{14}$

Comment

Step 8 of 32

Select $\mu^{(1)} = q + \left(-\frac{1}{y^{(1)}_1}\right)$ or $\mu^{(1)} = q + \left(\frac{1}{y^{(1)}_1}\right)$ so that, $\mu^{(1)}$ and q are close to each other.

Find $\mu^{(1)}$ and $x^{(1)} = \left(\frac{y^{(1)}}{y^{(1)}_1}\right) = \left(\frac{y^{(1)}}{\left\|y^{(1)}\right\|_\infty}\right)$ in continuation of the above maple commands.

```
> μ1 := evalf(q + (-1/Norm(y1, infinity)))
```

$\mu 1 := 1.307692308$

```
> x1 := (y1/Norm(y1, infinity))
```

$$x1 := \begin{bmatrix} -\frac{1}{13} \\ 1 \\ -\frac{8}{13} \end{bmatrix}$$

This maple output is the first iteration.

Comment

Step 9 of 32

Determine the second iteration in continuation of the above maple commands.

Find $y^{(2)} = (A-qI)^{-1} x^{(1)}$ and $\left\|y^{(2)}\right\|_\infty = y^{(2)}_1$ in continuation of the above maple commands.

```
> y2 := (A - q·i)^(-1).x1
```

$$y2 := \begin{bmatrix} \frac{57}{182} \\ -\frac{237}{182} \\ \frac{102}{91} \end{bmatrix}$$

```
> Norm(y2, infinity)
```

$\frac{237}{182}$

Comment

Step 10 of 32

Select $\mu^{(2)} = q + \left(-\frac{1}{y^{(2)}_1}\right)$ or $\mu^{(2)} = q + \left(\frac{1}{y^{(2)}_1}\right)$ so that, $\mu^{(2)}$ and q are close to each other.

Find $\mu^{(2)}$ and $x^{(2)} = \left(\frac{y^{(2)}}{y^{(2)}_1}\right) = \left(\frac{y^{(2)}}{\left\|y^{(2)}\right\|_\infty}\right)$ in continuation of the above maple commands.

```
> μ2 := evalf(q + (-1/Norm(y2, infinity)))
```

$\mu 2 := 0.8987341772$

```
> x2 := (y2/Norm(y2, infinity))
```

$$x2 := \begin{bmatrix} \frac{19}{79} \\ -1 \\ [illegible] \end{bmatrix}$$

This maple output is the second iteration.

Comment

Step 11 of 32

Determine the third iteration in continuation of the above maple commands.

[illegible]

Comment

Step 12 of 32

(b)

Determine the first three iterations obtained by the Inverse Power method to the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ with $x^{(0)} = (-1,0,1)^t$ by using Maple technology as shown below.

Input the given matrix A and the initial vector $x^{(0)}$

```
> with(LinearAlgebra) :
> A := Matrix([[1, 1, 1], [1, 1, 0], [1, 0, 1]])
```

$$A := \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

```
> x0 := ⟨-1, 0, 1⟩
```

$$x0 := \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

Find $q = \left\{ \frac{\left(x^{(0)}\right)^t A x^{(0)}}{\left(x^{(0)}\right)^t x^{(0)}} \right\}$ in continuation of the above maple commands.

```
> w := Transpose(x0)
```

$w := [\,-1 \;\; 0 \;\; 1\,]$

```
> q := w.A.x0 / w.x0
```

$q := 0$

Comment

Step 13 of 32

Find $y^{(1)} = (A-qI)^{-1} x^{(0)}$ and $\left\|y^{(1)}\right\|_\infty = y^{(1)}_1$ in continuation of the above maple commands.

```
> i := IdentityMatrix(3)
```

$$i := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

```
> y1 := (A - q·i)^(-1).x0
```

$$y1 := \begin{bmatrix} 2 \\ -2 \\ -1 \end{bmatrix}$$

```
> Norm(y1, infinity)
```

2

Comment

Step 14 of 32

Select $\mu^{(1)} = q + \left(-\frac{1}{y^{(1)}_1}\right)$ or $\mu^{(1)} = q + \left(\frac{1}{y^{(1)}_1}\right)$ so that, $\mu^{(1)}$ and q are close to each other.

Find $\mu^{(1)}$ and $x^{(1)} = \left(\frac{y^{(1)}}{y^{(1)}_1}\right) = \left(\frac{y^{(1)}}{\left\|y^{(1)}\right\|_\infty}\right)$ in continuation of the above maple commands.

```
> μ1 := evalf(q + (-1/Norm(y1, infinity)))
```

$\mu 1 := -0.5000000000$

```
> x1 := (y1/Norm(y1, infinity))
```

$$x1 := \begin{bmatrix} 1 \\ -1 \\ -\frac{1}{2} \end{bmatrix}$$

This maple output is the first iteration.

Comment

Step 15 of 32

Determine the second iteration in continuation of the above maple commands.

Find $y^{(2)} = (A-qI)^{-1} x^{(1)}$ and $\left\|y^{(2)}\right\|_\infty = y^{(2)}_1$ in continuation of the above maple commands.

```
> y2 := (A - q·i)^(-1).x1
```

$$y2 := \begin{bmatrix} -\frac{5}{2} \\ \frac{3}{2} \\ 2 \end{bmatrix}$$

```
> Norm(y2, infinity)
```

$\frac{5}{2}$

Comment

Step 16 of 32

Select $\mu^{(2)} = q + \left(-\frac{1}{y^{(2)}_1}\right)$ or $\mu^{(2)} = q + \left(\frac{1}{y^{(2)}_1}\right)$ so that, $\mu^{(2)}$ and q are close to each other.

Find $\mu^{(2)}$ and $x^{(2)} = \left(\frac{y^{(2)}}{y^{(2)}_1}\right) = \left(\frac{y^{(2)}}{\left\|y^{(2)}\right\|_\infty}\right)$ in continuation of the above maple commands.

```
> μ2 := evalf(q + (-1/Norm(y2, infinity)))
```

$\mu 2 := -0.4000000000$

```
> x2 := (y2/Norm(y2, infinity))
```

$$x2 := \begin{bmatrix} -1 \\ \frac{3}{5} \\ \frac{4}{5} \end{bmatrix}$$

This maple output is the second iteration.

Comment

Step 17 of 32

Determine the third iteration in continuation of the above maple commands.

Find $y^{(3)} = (A-qI)^{-1} x^{(2)}$ and $\left\|y^{(3)}\right\|_\infty = y^{(3)}_1$ in continuation of the above maple commands.

```
> y3 := (A - q·i)^(-1).x2
```

$$y3 := \begin{bmatrix} \frac{12}{5} \\ -\frac{9}{5} \\ -\frac{8}{5} \end{bmatrix}$$

```
> Norm(y3, infinity)
```

$\frac{12}{5}$

Step 18 of 32

Select $\mu^{(3)} = q + \left(-\frac{1}{y_1^{(3)}}\right)$ or $\mu^{(3)} = q + \left(\frac{1}{y_1^{(3)}}\right)$ so that, $\mu^{(3)}$ and q are close to each other.

Find $\mu^{(3)}$ and $x^{(3)} = \left(\frac{y^{(3)}}{y_1^{(3)}}\right) = \left(\frac{y^{(3)}}{\|y^{(3)}\|_\infty}\right)$ in continuation of the above maple commands.

> $\mu 3 := evalf\left(q + \left(-\frac{1}{Norm(y3, infinity)}\right)\right)$

$\mu 3 := -0.4166666667$

> $x3 := evalf\left(\frac{y3}{Norm(y3, infinity)}\right)$

$$x3 := \begin{bmatrix} 1. \\ -0.7500000000 \\ -0.6666666667 \end{bmatrix}$$

Thus, the approximate eigenvalue and the approximate eigenvector are,

$$\boxed{\mu^{(3)} = -0.4166666667,\ x^{(3)} = (1, -0.75, -0.6666666667)^t}$$

Comment

Step 19 of 32

(c)

Determine the first three iterations obtained by the Inverse Power method to the matrix

$A = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 4 & -2 \\ 0 & -1 & 2 \end{bmatrix}$ with $x^{(0)} = (-1, 2, 1)^t$ by using Maple technology as shown below.

Input the given matrix A and the initial vector $x^{(0)}$

> $with(LinearAlgebra):$

> $A := Matrix([[1,-1,0],[-2,4,-2],[0,-1,2]])$

$$A := \begin{bmatrix} 1 & -1 & 0 \\ -2 & 4 & -2 \\ 0 & -1 & 2 \end{bmatrix}$$

> $x0 := \langle -1, 2, 1 \rangle$

$$x0 := \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$$

Comment

Step 20 of 32

Find $q = \left\{\frac{\left(x^{(0)}\right)^t A x^{(0)}}{\left(x^{(0)}\right)^t x^{(0)}}\right\}$ in continuation of the above maple commands.

> $w := Transpose(x0)$

$w := \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}$

> $q := \frac{w.A.x0}{w.x0}$

$q := \frac{19}{6}$

> $evalf(q)$

3.166666667

Comment

Step 21 of 32

Find $y^{(1)} = (A - qI)^{-1} x^{(0)}$ and $\|y^{(1)}\|_\infty = y_1^{(1)}$ in continuation of the above maple commands.

> $i := IdentityMatrix(3)$

$$i := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

> $y1 := (A - q \cdot i)^{-1} x0$

$$y1 := \begin{bmatrix} \frac{114}{379} \\ \frac{132}{379} \\ -\frac{438}{379} \end{bmatrix}$$

> $Norm(y1, infinity)$

$\frac{438}{379}$

Comment

Step 22 of 32

Select $\mu^{(1)} = q + \left(-\frac{1}{y_1^{(1)}}\right)$ or $\mu^{(1)} = q + \left(\frac{1}{y_1^{(1)}}\right)$ so that, $\mu^{(1)}$ and q are close to each other.

Find $\mu^{(1)}$ and $x^{(1)} = \left(\frac{y^{(1)}}{y_1^{(1)}}\right) = \left(\frac{y^{(1)}}{\|y^{(1)}\|_\infty}\right)$ in continuation of the above maple commands.

> $\mu 1 := evalf\left(q + \left(-\frac{1}{Norm(y1, infinity)}\right)\right)$

$\mu 1 := 2.301369863$

> $x1 := \left(\frac{y1}{Norm(y1, infinity)}\right)$

$$x1 := \begin{bmatrix} \frac{19}{73} \\ \frac{22}{73} \\ -1 \end{bmatrix}$$

Determine the second iteration in continuation of the above maple commands.

Find $y^{(2)} = (A - qI)^{-1} x^{(1)}$ and $\|y^{(2)}\|_\infty = y_1^{(2)}$ in continuation of the above maple commands.

> $y2 := (A - q \cdot i)^{-1} x1$

$$y2 := \begin{bmatrix} -\frac{49278}{138335} \\ \frac{70764}{138335} \\ \frac{57918}{138335} \end{bmatrix}$$

> $Norm(y2, infinity)$

$\frac{70764}{138335}$

Comment

Step 23 of 32

Select $\mu^{(2)} = q + \left(-\frac{1}{y_1^{(2)}}\right)$ or $\mu^{(2)} = q + \left(\frac{1}{y_1^{(2)}}\right)$ so that, $\mu^{(2)}$ and q are close to each other.

Find $\mu^{(2)}$ and $x^{(2)} = \left(\frac{y^{(2)}}{y_1^{(2)}}\right) = \left(\frac{y^{(2)}}{\|y^{(2)}\|_\infty}\right)$ in continuation of the above maple commands.

> $\mu 2 := evalf\left(q + \left(-\frac{1}{Norm(y2, infinity)}\right)\right)$

$\mu 2 := 1.211788480$

> $x2 := \left(\frac{y2}{Norm(y2, infinity)}\right)$

$$x2 := \begin{bmatrix} -\frac{8213}{11794} \\ 1 \\ \frac{9653}{11794} \end{bmatrix}$$

This maple output is the second iteration.

Comment

Step 24 of 32

Determine the third iteration in continuation of the above maple commands.

Find $y^{(3)} = (A - qI)^{-1} x^{(2)}$ and $\|y^{(3)}\|_\infty = y_1^{(3)}$ in continuation of the above maple commands.

> $y3 := (A - q \cdot i)^{-1} x2$

$$y3 := \begin{bmatrix} \frac{3235377}{11174815} \\ \frac{771834}{11174815} \\ -\frac{8501187}{11174815} \end{bmatrix}$$

> $Norm(y3, infinity)$

$\frac{8501187}{11174815}$

Comment

Step 25 of 32

Select $\mu^{(3)} = q + \left(-\frac{1}{y_1^{(3)}}\right)$ or $\mu^{(3)} = q + \left(\frac{1}{y_1^{(3)}}\right)$ so that, $\mu^{(3)}$ and q are close to each other.

Find $\mu^{(3)}$ and $x^{(3)} = \left(\frac{y^{(3)}}{y_1^{(3)}}\right) = \left(\frac{y^{(3)}}{\|y^{(3)}\|_\infty}\right)$ in continuation of the above maple commands.

> $\mu 3 := evalf\left(q + \left(-\frac{1}{Norm(y3, infinity)}\right)\right)$

$\mu 3 := 1.852166115$

> $x3 := evalf\left(\frac{y3}{Norm(y3, infinity)}\right)$

$$x3 := \begin{bmatrix} 0.3805794414 \\ 0.09079132126 \\ -1. \end{bmatrix}$$

Thus, the approximate eigenvalue and the approximate eigenvector are,

$$\boxed{\mu^{(3)} = 1.852166115,\ x^{(3)} = (0.3805794414, 0.09079132126, -1)^t}$$

(d)

Determine the first three iterations obtained by the Inverse Power method to the matrix

$A = \begin{bmatrix} 4 & 1 & 1 & 1 \\ 1 & 3 & -1 & 1 \\ 1 & -1 & 2 & 0 \\ 1 & 1 & 0 & 2 \end{bmatrix}$ with $x^{(0)} = (1, -2, 0, 3)^t$ by using Maple technology as shown below.

Input the given matrix A and the initial vector $x^{(0)}$

> $with(LinearAlgebra):$

> $A := Matrix([[4,1,1,1],[1,3,-1,1],[1,-1,2,0],[1,1,0,2]])$

$$A := \begin{bmatrix} 4 & 1 & 1 & 1 \\ 1 & 3 & -1 & 1 \\ 1 & -1 & 2 & 0 \\ 1 & 1 & 0 & 2 \end{bmatrix}$$

> $x0 := \langle 1, -2, 0, 3 \rangle$

$$x0 := \begin{bmatrix} 1 \\ -2 \\ 0 \\ 3 \end{bmatrix}$$

Comment

Step 26 of 32

Find $q = \left\{\frac{\left(x^{(0)}\right)^t A x^{(0)}}{\left(x^{(0)}\right)^t x^{(0)}}\right\}$ in continuation of the above maple commands.

> $w := Transpose(x0)$

$w := \begin{bmatrix} 1 & -2 & 0 & 3 \end{bmatrix}$

> $q := \frac{w.A.x0}{w.x0}$

$q := \frac{12}{7}$

> $evalf(q)$

1.714285714

Comment

Step 27 of 32

Find $y^{(1)} = (A - qI)^{-1} x^{(0)}$ and $\|y^{(1)}\|_\infty = y_1^{(1)}$ in continuation of the above maple commands.

> $i := IdentityMatrix(4)$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

> $y1 := (A - q \cdot i)^{-1} x0$

$$\begin{bmatrix} \frac{6545}{2542} \\ \frac{3353}{1271} \\ \frac{1127}{5084} \\ -\frac{39375}{5084} \end{bmatrix}$$

> $Norm(y1, infinity)$

$\frac{39375}{5084}$

Comment

Step 28 of 32

Select $\mu^{(1)} = q + \left(-\frac{1}{y_1^{(1)}}\right)$ or $\mu^{(1)} = q + \left(\frac{1}{y_1^{(1)}}\right)$ so that, $\mu^{(1)}$ and q are close to each other.

Find $\mu^{(1)}$ and $x^{(1)} = \left(\frac{y^{(1)}}{y_1^{(1)}}\right) = \left(\frac{y^{(1)}}{\|y^{(1)}\|_\infty}\right)$ in continuation of the above maple commands.

> $\mu 1 := evalf\left(q + \left(-\frac{1}{Norm(y1, infinity)}\right)\right)$

$\mu 1 := 1.585168254$

> $x1 := \left(\frac{y1}{Norm(y1, infinity)}\right)$

$$x1 := \begin{bmatrix} -\frac{374}{1125} \\ -\frac{1916}{5625} \\ -\frac{161}{5625} \\ 1 \end{bmatrix}$$

This maple output is the first iteration.

Determine the second iteration in continuation of the above maple commands.

Find $y^{(2)} = (A - qI)^{-1} x^{(1)}$ and $\|y^{(2)}\|_\infty = y_1^{(2)}$ in continuation of the above maple commands.

Comment

Step 29 of 32

> $y2 := (A - q \cdot i)^{-1} x1$

$$y2 := \begin{bmatrix} -\frac{2321501}{2383125} \\ -\frac{2049761}{2383125} \\ \frac{7138957}{14298750} \\ \frac{13916959}{4766250} \end{bmatrix}$$

> $Norm(y2, infinity)$

$\frac{13916959}{4766250}$

Comment

Step 30 of 32

Select $\mu^{(2)} = q + \left(-\frac{1}{y_1^{(2)}}\right)$ or $\mu^{(2)} = q + \left(\frac{1}{y_1^{(2)}}\right)$ so that, $\mu^{(2)}$ and q are close to each other.

Find $\mu^{(2)}$ and $x^{(2)} = \left(\frac{y^{(2)}}{y_1^{(2)}}\right) = \left(\frac{y^{(2)}}{\|y^{(2)}\|_\infty}\right)$ in continuation of the above maple commands.

> $\mu 2 := evalf\left(q + \left(-\frac{1}{Norm(y2, infinity)}\right)\right)$

$\mu 2 := 1.371807878$

> $x2 := \left(\frac{y2}{Norm(y2, infinity)}\right)$

$$x2 := \begin{bmatrix} -\frac{663286}{1988137} \\ -\frac{585646}{1988137} \\ \frac{1019851}{5964411} \\ 1 \end{bmatrix}$$

This maple output is the second iteration.

Comment

Step 31 of 32

Determine the third iteration in continuation of the above maple commands.

Find $y^{(3)} = (A - qI)^{-1} x^{(2)}$ and $\|y^{(3)}\|_\infty = y_1^{(3)}$ in continuation of the above maple commands.

> $y3 := (A - q \cdot i)^{-1} x2$

$$y3 := \begin{bmatrix} \frac{2778606334}{2526922127} \\ \frac{5698833413}{7580766381} \\ -\frac{4692642388}{7580766381} \\ -\frac{22588601119}{7580766381} \end{bmatrix}$$

> $Norm(y3, infinity)$

$\frac{22588601119}{7580766381}$

Comment

Step 32 of 32

Select $\mu^{(3)} = q + \left(-\frac{1}{y_1^{(3)}}\right)$ or $\mu^{(3)} = q + \left(\frac{1}{y_1^{(3)}}\right)$ so that, $\mu^{(3)}$ and q are close to each other.

Find $\mu^{(3)}$ and $x^{(3)} = \left(\frac{y^{(3)}}{y_1^{(3)}}\right) = \left(\frac{y^{(3)}}{\|y^{(3)}\|_\infty}\right)$ in continuation of the above maple commands.

> $\mu 3 := evalf\left(q + \left(-\frac{1}{Norm(y3, infinity)}\right)\right)$

$\mu 3 := 1.378684304$

Continuation of the above

> $x3 := evalf\left(\frac{y3}{Norm(y3, infinity)}\right)$

$$x3 := \begin{bmatrix} 0.3690276772 \\ 0.2522880183 \\ -0.2077438246 \\ -1. \end{bmatrix}$$

Thus, the approximate eigenvalue and the approximate eigenvector are,

$$\boxed{\mu^{(3)} = 1.378684304,\ x^{(3)} = (0.3690276772, 0.2522880183, -0.2077438246, -1)^t}$$

Comment